Similarity Concept in Theory Lecturing: Application to Transportation Studies

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Abstract

In this paper, a similarity concept is proposed to improve student understanding on difficult and complicated engineering theory. The planned application of this approach is for the Transportation Studies module (CSE6004-A) at School of Engineering, University of Bradford, United Kingdom. In the module, noise induced by road transport and vehicles are taught in depth, where the proposed teaching method will be applied to aid student understanding on the numerical concept of the vibration effect and noise on vehicle braking system. As part of the module planning, the full numerical solution of brake judder/vibration effect, which includes shaking (forced vibration) and nibbling (torsional vibration) effects will be introduced to students where similarity concept will be adapted in its teaching. The successfully applied concept will also be able to utilize by other engineering teaching and modules.

Keywords: Similarity Concept; Theory Lecturing; Transportation Studies; Vehicle Braking; Noise

1. Introduction of Educational Materials and Plan

Similarity concept has been a key foundation to many engineering theories for centuries. In this concept, the difficult theories or numerical derivations are reduced to an adaptable form of formulas for simplifying them into a useful engineering calculation approach. One of the examples is the Bernoulli's theorem[1], where different quantities are converted into the same considering basic (with same head unit) to allow the comparison of different quantity sets and to make the complicated engineering problem comprehensible to engineers and students.

In this study, the similarity concept is applied to plan the Transportation Studies module under Civil Engineering stream. The module introduces to the Stage 3/Level 6 students (equivalent to Bachelor of Engineering Degree's Final Year) the advanced concepts of road transport. Due to the final year level, the course is planned to be improved to include complicated numerical derivation and theory of transportation. There is an emphasis in the module on the road and vehicle noise, and their contribution towards transportation inefficiency, hence it is planned that derivation of vibration (or noise) induced numerical model will be a necessary concept for the students to understand.

In this vibration concept for vehicle, the braking induced vibrations are usually subjected to different shaking (forced vibration) and nibbling (torsional vibration) effects, which are regarded as brake judder. In university level, those brake judder's vibrations are usually taught as separate concepts, which can be confusing to students as both theories are difficult and complicated to understand. This vibration concept is also crucial in modern days' transportation studies due to the significant increase in vehicle top speed and the braking power aligns with such top speed.

By definition, a braking system is functioning by transferring the kinetic energy of rotating disc irreversibly into heat, which is then transferred to the surrounding environment. The rubbing surfaces allow force and torque to be developed in between, which caused by their different angular speed. Brake judder is a critical drawback for any braking system as it could delay and reduce the inputted braking effort. In terms of teaching, a system of equivalent spring and damper could be used to represent the braking system, as showing in Figure 1. A harmonic excitation force represented by F_N is used as the source of vibration. F_o is the normal force applied to the brake pad, and A is the force amplitude of the brake judder. ω and t are angular velocity and time respectively.

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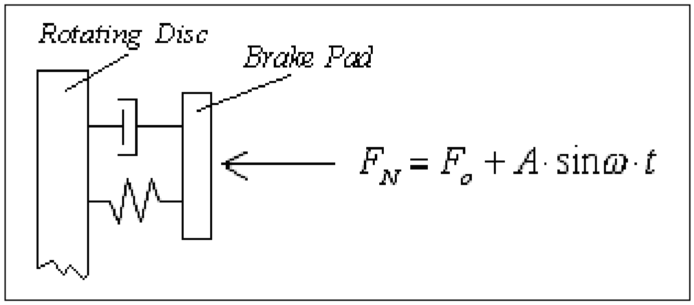

Figure 1. Schematic of the braking system

According to Newton's Second Law at equation (1), the sum of all forces acting on a system in a certain direction is equaled to the product of the total mass for that system, m, and the acceleration, a_x, in that particular direction. It will become the fundamental of this taught numerical modeling.

$$\sum F_x = m \cdot a_x \tag{1}$$

The main ideas in this braking system vibration analysis are shown as below,

1. The excitation force of the shaking and nibbling is in the harmonic form and varies with time. [2]
2. The transient period to achieve pro-steady state of the vibrations is going to die-out very fast in the earlier state of braking. [2, 3]
3. All the variables, like velocity and acceleration, during braking event are time-dependent. [2, 3]

In the outcome of this study, the similarity concept will be applied into the teaching of different vibration models in vehicle transportation. This paper will also discuss how this similarity concept can facilitate the student learning by making the teaching more effective. In doing so, the force and torsional vibration models derived will be compared for their similarity in order to promote deep understanding of the taught models among students.

1.1. ***Numerical Modeling of Shaking (Forced Vibration)***

In the forced vibration numerical modeling, the shaking of the brake pad is included with a small displacement shift, Δ, that caused by the brake judder phenomenon and the total equation of motion for the shaking on a brake pad is

$$(F_o + A\sin\omega \cdot t) - k(x + \Delta) - c(\dot{x} + \dot{\Delta}) = m(\ddot{x} + \ddot{\Delta}) \tag{2}$$

where k and c are stiffness and damping coefficients of the linear vibration respectively, which displaced by a distance of x.

We can assume that the pad motion is solely caused by F_o that initiated into the braking system, as suggested by[2], hence for shaking effect of the brake judder can be written as

$$m\ddot{\Delta}+c\dot{\Delta}+k\Delta = A\sin\omega\cdot t \tag{3}$$

During braking, the shaking consists two types of period, transient and steady state periods. As a result, the solution of Δ becomes

$$\Delta = \Delta_c(t)+\Delta_p(t) \tag{4}$$

where Δ_c and Δ_p represent the transient and steady state solution of Δ, respectively.

1.1.1. Analysis Δ_c

The solution of Δ could be pre-assumed to be an exponential expression, in consistence with the input harmonic excitation force.[4] By using this assumption, a pre-solution of Δ_c and its derivatives could be predicted as follows

$$\Delta_c = e^{p\cdot t},\ \dot{\Delta}_c = p\cdot e^{p\cdot t},\ \text{and}\ \ddot{\Delta}_c = p^2\cdot e^{p\cdot t} \tag{5}$$

where p is the proposed solution? For the transient period solution, since it needs only to satisfy the differential equation condition, its characteristic equation can be deduced to

$$m\ddot{\Delta}_c + c\dot{\Delta}_c + k\Delta_c = 0 \tag{6}$$

After substituting equation (5) into (6), the formula below can be obtained, where the Δ_c-dependent equation is now shifted to the p-dependent, provided Δ_c-solution is never going to be zero.

$$m\cdot p^2 + c\cdot p + k = 0 \tag{7}$$

where

$$p = -\frac{c}{2m} \pm \frac{\sqrt{c^2-4mk}}{2m} \tag{8}$$

By using the formulae $\xi = \dfrac{c}{2\cdot\sqrt{m\cdot k}}$ and $\omega_n = \sqrt{\dfrac{k}{m}}$ into equation (8), the below equation can be obtained

$$p = -\xi\omega_n \pm \omega_n\cdot\sqrt{\xi^2-1} \tag{9}$$

where ξ is the damping ratio, and ω_n is the natural frequency of brake pad? However, there are three possible cases of ξ for the solution of Δ_c, which are listed at sub-sections below.

1.1.2. Critical Damped Case ($\xi^2 = 1$)

Since $\xi = 1$ and damping ratio cannot accept any negative value, equation (9) becomes

$$p = -\omega_n \tag{10}$$

therefore

$$\Delta_c = (B + C_1 \cdot t) \cdot \exp(-\omega_n \cdot t) \tag{11}$$

where B and C_1 are both constants that could be found using the initial condition.

1.1.3. Over-damped Case ($\xi^2 > 1$):

Since $\xi > 1$, $\sqrt{\xi^2 - 1}$ will be definitely a real value. Hence, equation (9) becomes

$$p = -\xi\omega_n \pm \omega_n \sqrt{\xi^2 - 1} \tag{12}$$

therefore

$$\Delta_c = B \cdot \exp[(-\xi + \sqrt{\xi^2 - 1})\omega_n \cdot t] + C_1 \cdot \exp[(-\xi - \sqrt{\xi^2 - 1})\omega_n \cdot t] \tag{13}$$

1.1.4. Under-damped Case ($\xi^2 < 1$)

Since $\xi < 1$, $\sqrt{\xi^2 - 1}$ will be definitely an imaginary value. Hence, (9) becomes

$$p = -\xi\omega_n \pm i\omega_n \sqrt{1 - \xi^2} \tag{14}$$

therefore

$$\Delta_c = \exp(-\xi\omega_n) \cdot [B \cdot \exp(i\omega_n \sqrt{1 - \xi^2} \cdot t) + C_1 \cdot \exp(-i\omega_n \sqrt{1 - \xi^2} \cdot t)] \tag{15}$$

1.2 Analysis of Δ_p

For analyzing Δ_p, a pre-solution and its derivatives are proposed in equation below

$$\Delta_p = D \cdot e^{i\omega t},\ \dot{\Delta}_p = i\omega D \cdot e^{i\omega t},\ \text{and}\ \ddot{\Delta}_p = -\omega^2 D \cdot e^{i\omega t} \tag{16}$$

where D is an assumed multiplication factor to exponential solution? Since Δ_p needs to satisfy both the differential equation and initial conditions of the pre-solution at equation (16), its characteristic equation should be written as

$$m\ddot{\Delta}_p + c\dot{\Delta}_p + k\Delta_p = A \cdot e^{i\omega t} \tag{17}$$

If we substitute equation (16) into (17), the formula below can be obtained by assuming $\Delta_p \neq 0$.

$$-D \cdot m\omega^2 + iD \cdot c\omega + D \cdot k = A \tag{18}$$

where

$$D = \frac{A(k - m\omega^2)}{(k - m\omega^2)^2 + (c\omega)^2} - i\frac{A(c\omega)}{(k - m\omega^2)^2 + (c\omega)^2} \tag{19}$$

By transforming the linear form of equation (19) into the angular form, we can deduce

$$D = U + iV = E \cdot e^{i\theta} \tag{20}$$

where $U = \frac{A(k - m\omega^2)}{(k - m\omega^2)^2 + (c\omega)^2}$, $V = -\frac{A(c\omega)}{(k - m\omega^2)^2 + (c\omega)^2}$, $E = \sqrt{U^2 + V^2}$, and, $\theta = \tan^{-1}\frac{V}{U}$

If we substitute equation (20) into (16), the pre-solution becomes

$$\Delta_p = E \cdot e^{i(\omega t + \theta)} \tag{21}$$

By referring to the characteristic equation of shaking in (3), we know that only imaginary part of the pre-solution at equation (21) will be considered in this part of the solution, hence

$$\begin{aligned} \Delta_p &= E \cdot \sin(\omega t + \theta) \\ &= \frac{A}{\sqrt{(k - m\omega^2)^2 + (c\omega)^2}} \cdot \sin\left[\omega t + \tan^{-1}\frac{c\omega}{k - m\omega^2}\right] \end{aligned} \tag{22}$$

1.3. Final Forced Vibration Model

From equation (4), we know that the expression of Δ is the sum of Δ_c and Δ_p. However, the transient solution of shaking usually die-out very soon after the starting of braking event.[2,3] Hence, the final solution of shaking should take the form of Δ_p and becomes

$$\Delta = \frac{A}{\sqrt{(k - m\omega^2)^2 + (c\omega)^2}} \cdot \sin\left[\omega t + \tan^{-1}\frac{c\omega}{k - m\omega^2}\right] \tag{23}$$

2. Numerical Modeling of Nibbling (Torsional Vibration)

In the numerical modeling of nibbling, the equation of angular motion used for brake pad can be written as [2]

$$J_p \cdot (\ddot{\Phi}_p + \ddot{\phi}_p) + C \cdot (\dot{\Phi}_p + \dot{\phi}_p) + K \cdot (\Phi_p + \phi_p) = M_t \quad (24)$$

where J, C, and K, are the inertia moment, damping and stiffness coefficients for the torsional vibration respectively. Φ_p and ϕ_p are the angular motion and vibration for the brake pad respectively, and M_t is the total brake torque.

However, in order to investigate the brake judder phenomenon, the total brake torque has been converted into a function of mean brake torque level, M_o, relative brake torque variation, ε, the angular motion, Φ_D, and angular vibration, ϕ_D, of the rotating disc[2]

$$M_t = M_o + \varepsilon \cdot M_o \cdot \sin[\Phi_D(t) + \phi_D(t)] \quad (25)$$

Since the disc judder is very small due to its physical constraints and properties, it can be ignored to transform equation (25) to

$$M_t = M_o + \varepsilon \cdot M_o \cdot \sin[\Phi_D(t)] \quad (26)$$

By substituting equation (26) into (24), we are able to find out the effects of brake torque variation and mean brake torque toward the pad motion as follows

$$(J_p \ddot{\Phi}_p + C \dot{\Phi}_p + K\Phi_p) + (J_p \ddot{\phi}_p + C \dot{\phi}_p + K\phi_p) = M_o + \varepsilon \cdot M_o \cdot \sin[\Phi_D(t)] \quad (27)$$

Since the inputted M_o should be balanced up by the pad motion, what is left will be the pad judder

$$J_p \ddot{\phi}_p + C \dot{\phi}_p + K\phi_p = \varepsilon \cdot M_o \cdot \sin[\Phi_D(t)] \quad (28)$$

The t-dependent Φ_D can be related to the initial angular velocity, Ω_o, and angular acceleration, α, of the pad as showing below

$$\Phi_D(t) = \Omega_o \cdot t + \frac{1}{2} \cdot \alpha \cdot t^2 \quad (29)$$

The angular acceleration, α, can be ignored due to the transient period is much shorter than the steady state one, and hence equation (29) can be simplified to

$$\Phi_D(t) = \Omega_o \cdot t \tag{30}$$

Substitute equation (30) into (28), the characteristic equation becomes

$$J_p \ddot{\phi}_p + C\dot{\phi}_p + K\phi_p = \varepsilon \cdot M_o \cdot \sin(\Omega_o \cdot t) \tag{31}$$

By following the same procedures as numerical modelling of shaking from equation (3) to (23), the solution of the nibbling judder can be obtained as below, which represents the final proposed torsional vibration model.

$$\phi_p = \frac{\varepsilon \cdot M_o}{\sqrt{[K - J_p \cdot (\Omega_o)^2]^2 + (C \cdot \Omega_o)^2}} \cdot \sin\left[\Omega_o \cdot t + \tan^{-1}\frac{C \cdot \Omega_o}{K - J_p \cdot (\Omega_o)^2}\right] \tag{32}$$

3. Similarity Concept

In comparison, the torsional vibration in braking represents a much advance theoretical concept compared to the forced vibration. The concept for brake nibbling has only been developed since about two decades ago. [4] Hence from the students' viewpoint, the forced vibration will be a more familiar concept compared to its torsional counterpart.

In delivering the lecture, the strategy employed will be to emphasize the derivation of forced vibration and simplify the torsional vibration part. The similarity concept is utilized for this simplification. From equation (31) to (32) above, there supposed to be a full and long derivation similar to that from equation (3) to (23); but it has been simplified. From the understanding obtained by forced vibration derivation, students will have the needed knowledge to work out final torsional vibration model using the exact same procedures. And by utilizing this similarity concept, the benefits below can be obtained

1. Shortening of the lecturing period and effort on complicated theoretical derivation.
2. Encouraging students to work independently to solve the torsional vibration solution without disadvantage them from foreseeing the solution.
3. Increasing the interest of students during the complicated theoretical derivations as they are required to think of the derivation using the provided guideline from forced vibration derivation procedures.
4. Improving students' focus during the lecture. If the derivation is too difficult the students will be discouraged; but if it is too easy or if every derivation steps are provided to them then they will not put any thinking into the topic. Hence the optimized procedures will be to encourage them for

trying but providing them with pre-solution procedure, like what is provided by the proposed similarity conceptual lecturing method.

4. Conclusion

A similarity concept has been proposed in teaching the complicated theoretical derivation/concept of engineering. This proposed concept will be utilized into a Transportation Studies module run for final year students of Bachelor in Engineering, where a full case of derivation using the brake vibration was discussed for detailing the application of the proposed concept. The concept can shorten lecture time while benefiting the students' learning. The final proposed concept is also applicable for other engineering module with similar type of theoretical teaching.

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